Imprecise Probability Analysis for Integrated Assessment of Climate Change

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Carnegie Mellon University, Pittsburgh

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Overview

1. Why imprecise probabilities?

2. Prototypical assessment of climate change with imprecise probabilities

3. Outlook: Research plans as Marie Curie fellow at CMU and PIK

Motivation

Epistemic uncertainty:

Lack of knowledge about system properties and processes

Lack of data about historical system development

Aleatory (“physical”) uncertainty:

Due to inherent variability in the system

Quantifying epistemic uncertainty in IA-models is a challenge

- difficult to measure (no frequency interpretation of probability!)
- But: demand for probabilities due to decision context

(Schneider, Nature 411, 2001; Dessai & Hulme, Tyndall Centre Working Paper 34, 2003)

Motivation

⇒ Application of the Bayesian approach to probability

1. Comparison of model hypotheses θ with observations y

   Likelihood function $L^y(\theta) := p(y|\theta)$

   (Classical statistics: Estimation of confidence regions for θ)

2. Estimation of uncertainty prior to observations

   Prior probability P: $\Sigma(\Omega) \rightarrow [0, 1]$ (Ω: space of parameters $\theta \in \Omega$
   $\Sigma(\Omega)$: event space $A \subseteq \Omega$)

3. Bayesian updating *(with information gained from observations)*

   Posterior probability P_y: $\Sigma(\Omega) \rightarrow [0, 1]$, $p_y(\theta|y) := L^y(\theta)\, p(\theta) / p(y)$

Motivation

Large uncertainty $\Rightarrow$ There is no unique prior probability that is compatible with the state of information

Weaker requirement: Imprecise probability

Generalization of the Bayesian approach ("Robust Bayes")

Describes sets of probabilities

Intermediate between interval uncertainty and probability

Research Question

Quantifying uncertainty in integrated assessment (IA) models by means of imprecise probabilities

- Can imprecise probabilities improve the evidential foundation on the (in some places very weak) state of information?
- Do numerical methods exist to process imprecise probabilities in an integrated assessment of climate change?

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- Model construction and comparison with observations

  Climate model $M(\theta)$, comparison with 20th century temperature record

- Estimating the uncertainty about model parameters θ

  Imprecise prior probability, Bayesian updating

- Processing the uncertainty for an assessment of climate change

  Imprecise probability for future global mean warming

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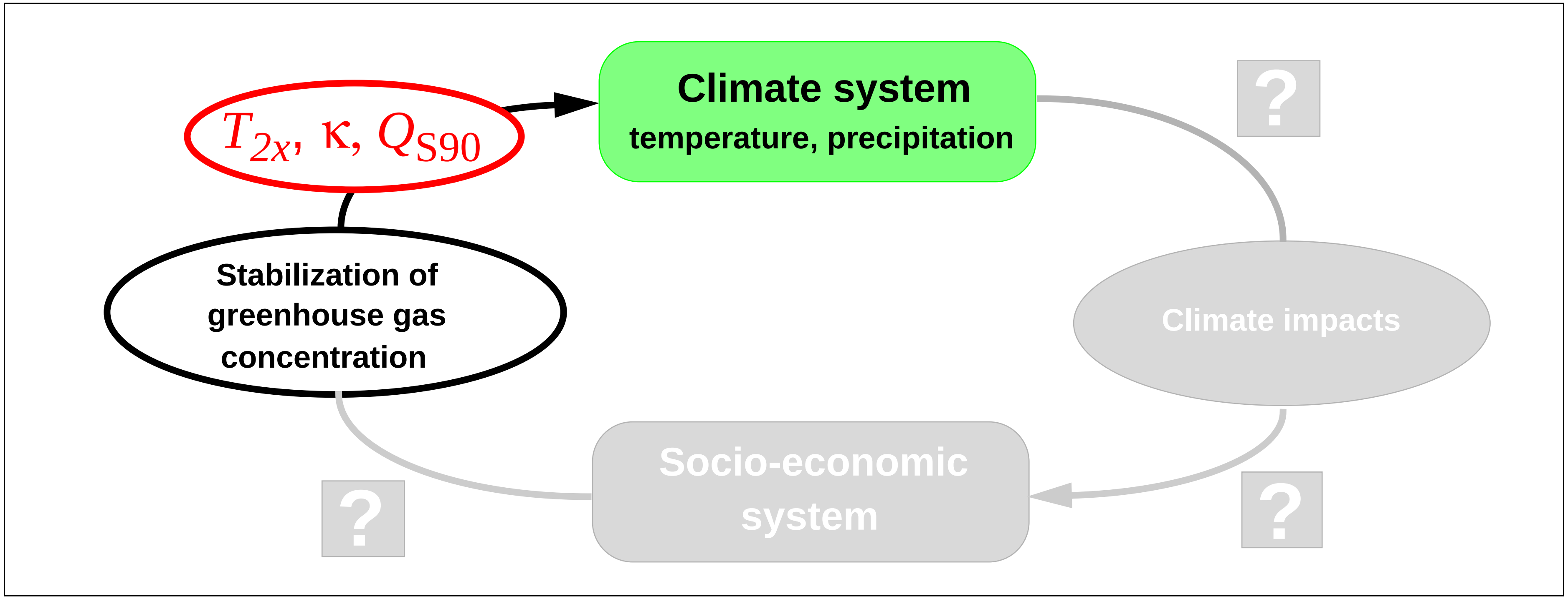
Definition of Uncertain Parameter Space



Key factors influencing the climate response (e.g., Forest et al., Science 295, 2002)

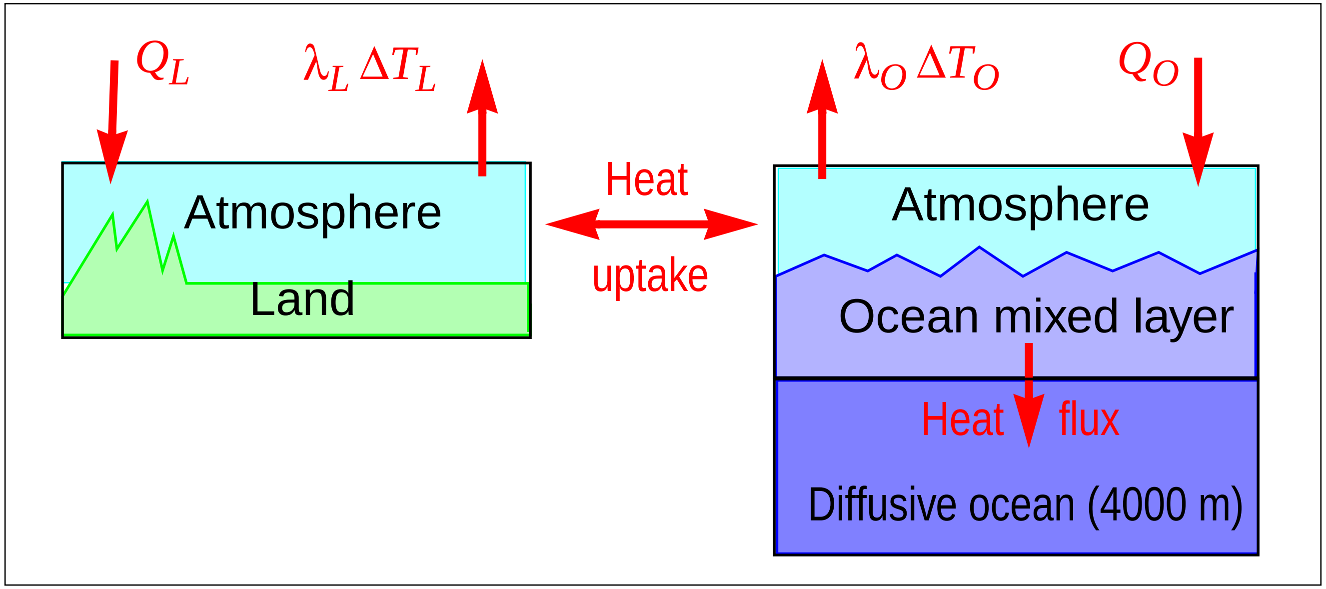
- climate sensitivity (temperature response to doubling of atmospheric CO_2: T_{2x})

- ocean heat uptake (effective vertical diffusivity: κ)

- sulphate aerosol cooling (radiative forcing in 1990: Q_{S90})

Climate Model

Energy balance model (EBM) with 1D-diffusive ocean



- Land-atm. box: $C_L\, dT_L/dt = Q_L(Q_{S90}) - \lambda_L(T_{2x})\, \Delta T_L - k(T_{2x}) / f_L\, (\Delta T_L - b\, \Delta T_O)$

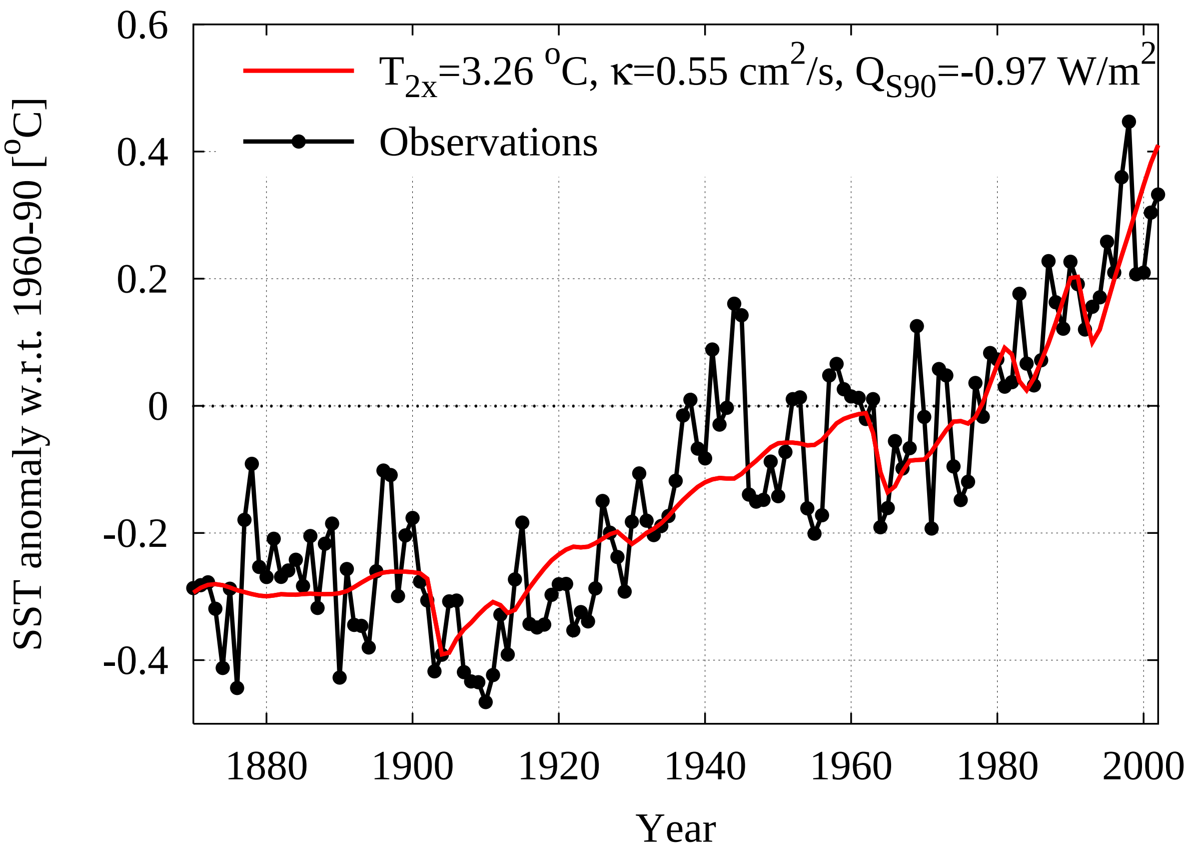
- Ocean-atm. box: $C_O\, dT_O/dt = Q_S(Q_{S90}) - \lambda_O(T_{2x})\, \Delta T_O - k(T_{2x}) / (1-f_L)\, (b\, \Delta T_O - \Delta T_L) - F_O(\kappa)$

- 1D-diffusive ocean: $\frac{\partial}{\partial t} T(z, t) = \kappa \frac{\partial^2}{\partial z^2} T(z, t)$, $T(0, t) = \Delta T_O(t)$, $\frac{\partial}{\partial z} T(z_B, t) = 0$

Natural Temperature Variability

Sea surface temperature (SST)

(HadSST data, Jones et al., J. Geoph. Res. 106, 2001)



Residual data - model: ENSO and weather-driven variability

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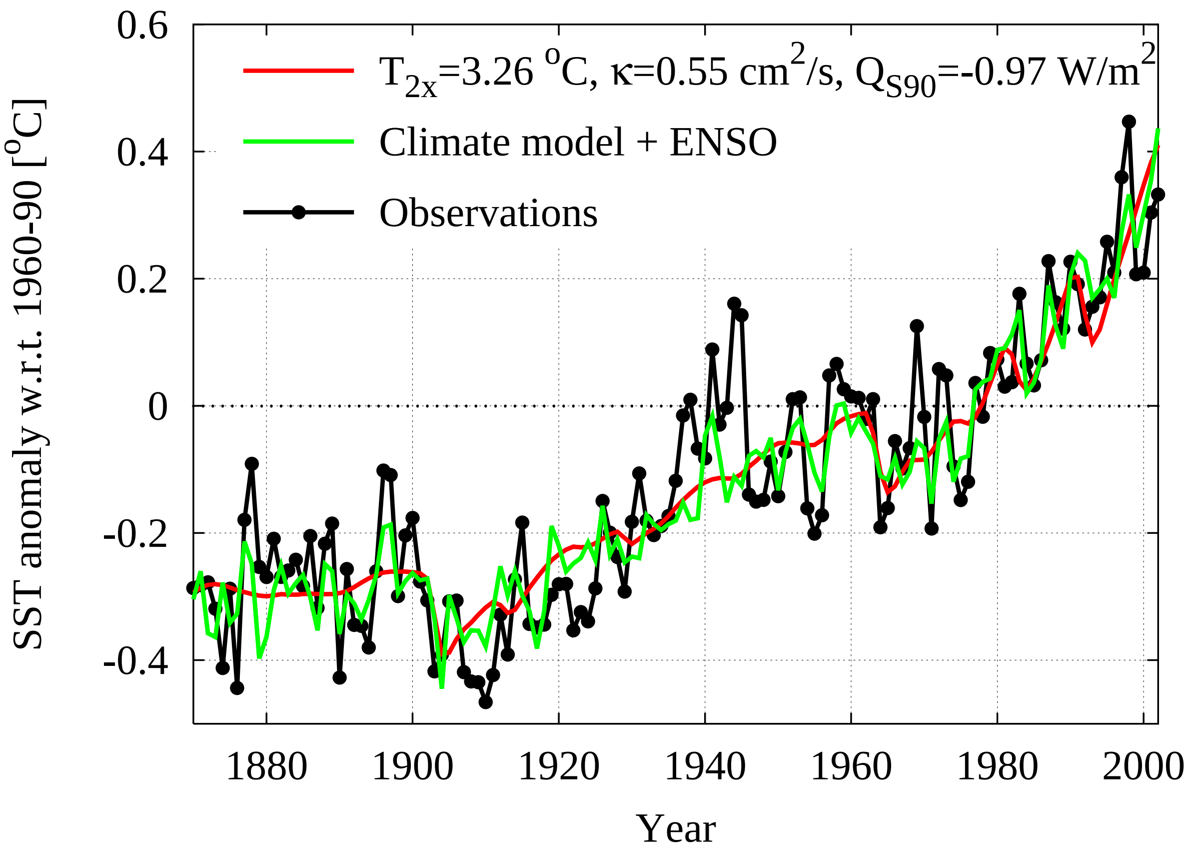


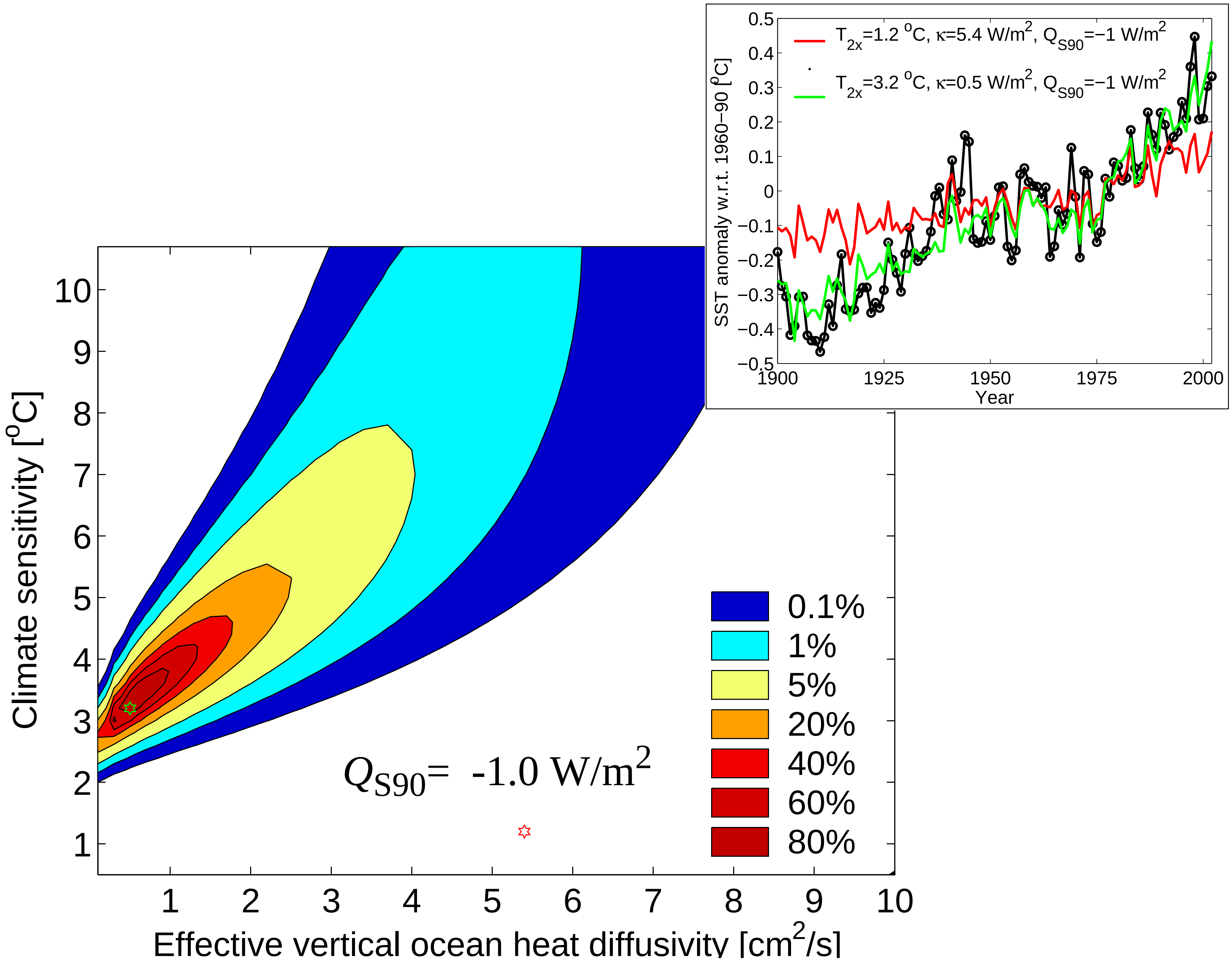
Residual data - model: ENSO and weather-driven variability

Definition of Likelihood Function

Likelihood function $L^y(\theta)$ $\Leftrightarrow$ Probability of residual being ...

SST: $r_{SST} = y_{SST} - T_O(T_{2x}, \kappa, Q_{S90}) - \beta_O\, SOI$ **AR(1)-process**

$\Delta \boldsymbol{T} = \boldsymbol{T_L} - \boldsymbol{T_O}$: $r_{\Delta T} = y_L - T_L(T_{2x}, \kappa, Q_{S90}) - \beta_L\, SOI - r_{SST}$ **white noise**



Climate sensitivity [^oC]
Effective vertical ocean heat diffusivity [cm^2/s]
Q_{S90}= -1.0 W/m^2
0.1%
1%
5%
20%
40%
60%
80%
SST anomaly w.r.t. 1960−90 [^oC]
Year
T_{2x}=1.2 ^oC, κ=5.4 W/m^2, Q_{S90}=−1 W/m^2
T_{2x}=3.2 ^oC, κ=0.5 W/m^2, Q_{S90}=−1 W/m^2

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Probability

Probability

Measure P: $\Sigma(\Omega_n) \rightarrow [0, 1]$

Possibility space $\Omega_n = \{a_1, .., a_n\}$

$\Rightarrow$ event space $\Sigma(\Omega_n)$

with 2^n events $A \subseteq \Omega_n$

Representation in terms of probability mass function:

$$\boldsymbol{p}: \Omega_n \rightarrow [0, 1], \quad P(A) = \sum_{x_i \in A} p_i$$

Imprecise Probability

Probability	Imprecise Probability
Single probability	Convex set of probabilities
Measure $P: \Sigma(\Omega_n) \to [0, 1]$ Possibility space $\Omega_n = \{a_1,..,a_n\}$ $\Rightarrow$ event space $\Sigma(\Omega_n)$ with 2^n events $A \subseteq \Omega_n$	Special case: interval probability Measure $\underline{P}: \Sigma(\Omega_n) \to [0, 1]$ $\underline{P}(A) \equiv inf_{P \in \Gamma} P(A)$
Representation in terms of probability mass function: $\boldsymbol{p}: \Omega_n \to [0, 1], \; P(A) = \sum_{x_i \in A} p_i$	Special case: belief (Dempster-Shafer theory) Representation in terms of Möbius inverse: $\mu: \Sigma(\Omega_n) \to [0, 1], \; \underline{P}(A) = \sum_{B \subseteq A} \mu(B)$

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Quantification of Prior Uncertainty

Conditions on suitable class of imprecise probability

1. Intuitively accessible (e.g., by means of expert elicitation):

Specification in terms of probability bounds $\underline{P}(A) \leq P(A) \leq \overline{P}(A)$

for a finite number of intuitively accessible events:

- atoms $A_1,\ldots, A_n$ of a partition of Ω

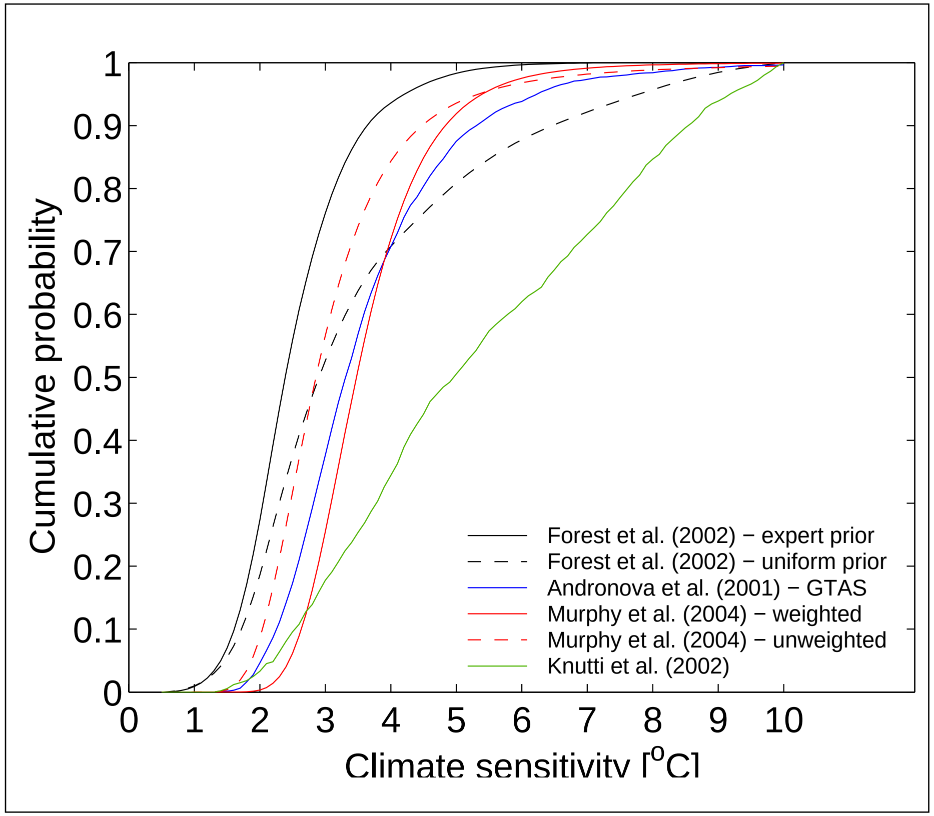
- "cumulative events" $(-\infty, \boldsymbol{x}_i],\quad x_1 < \ldots < x_k \in \Omega$

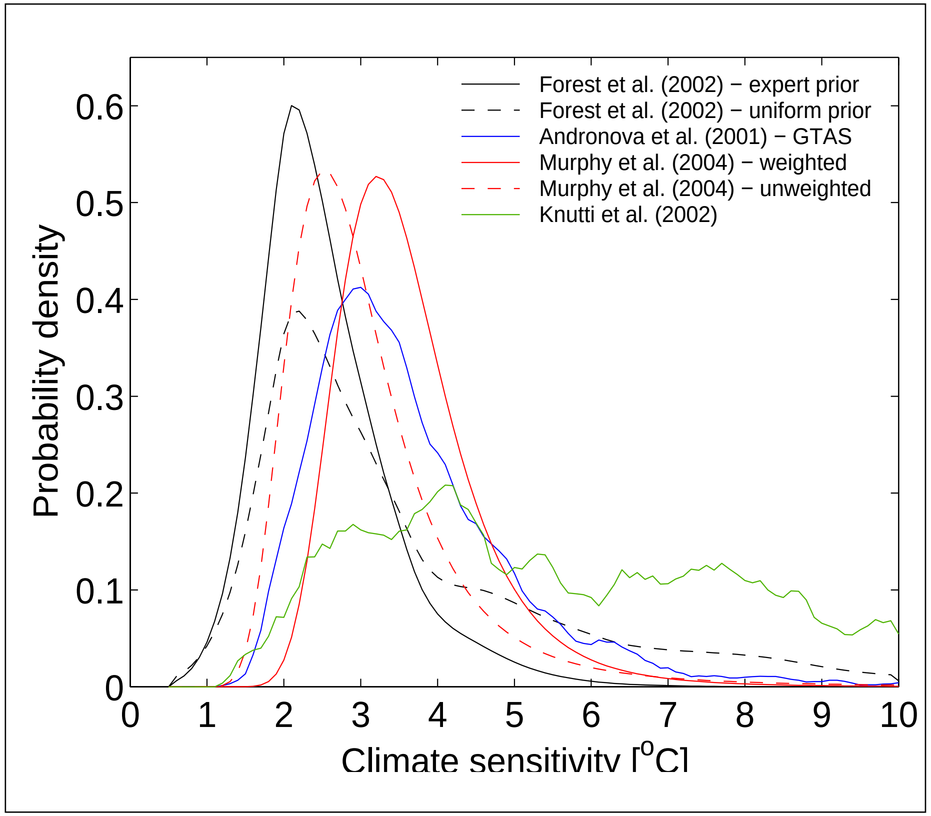
2. Numerically tractable:

Representation in terms of a "sparse" Möbius inverse,

i.e., small number $m \ll 2^n$ of focal elements B with $\mu(B) > 0$

Prior Uncertainty about Climate Sensitivity

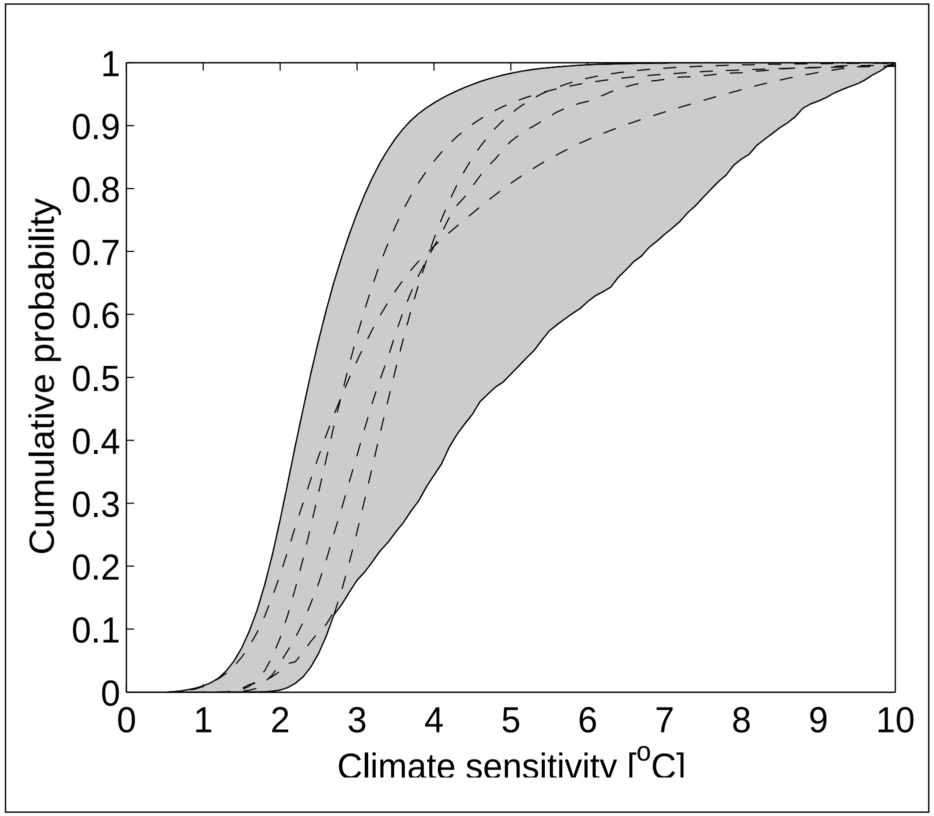




Prior Uncertainty about Climate Sensitivity

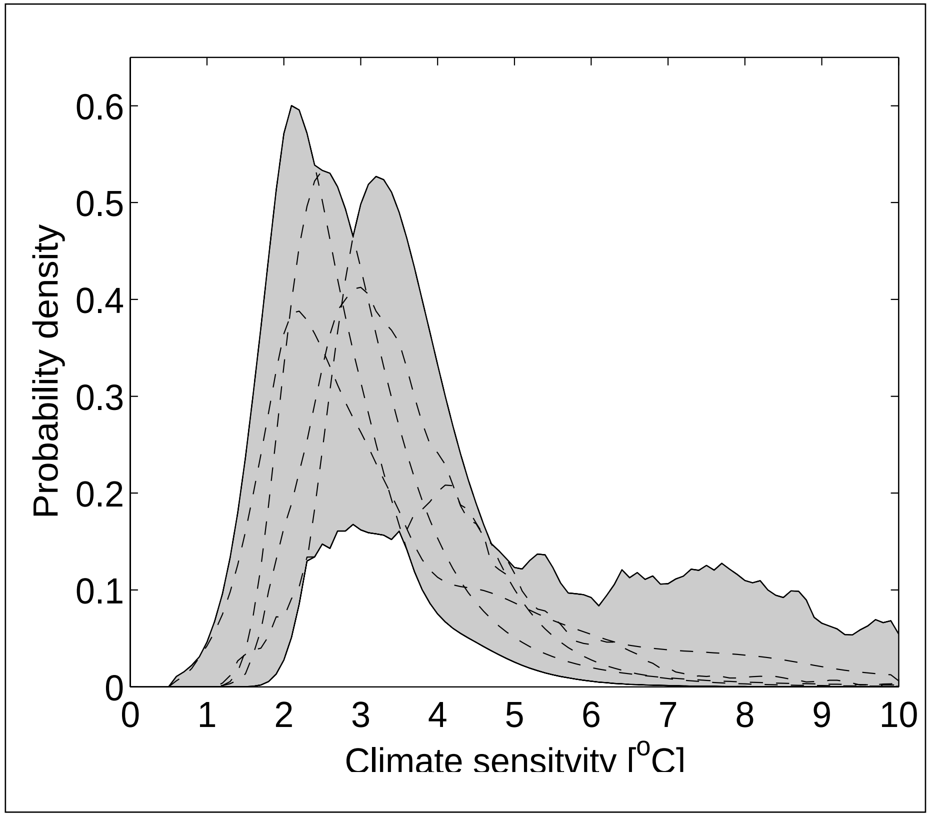
Distribution band

$$\Gamma(\underline{F}, \overline{F}) \equiv \{ P | \underline{F}(x) \le P(-\infty, x] \le \overline{F}(x) \}$$



Density band

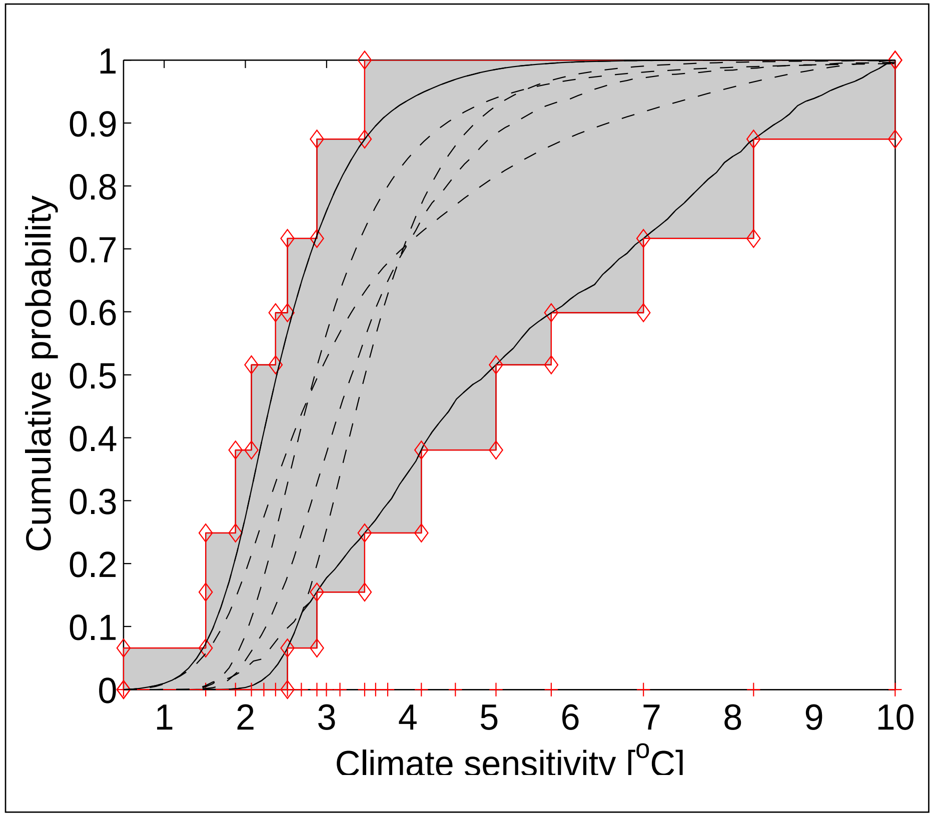
$$\Gamma(\underline{\rho}, \bar{\rho}) \equiv \{ P | \underline{\rho}(x) \le \rho(x) \le \bar{\rho}(x) \}$$



Prior Uncertainty about Climate Sensitivity

Distribution band

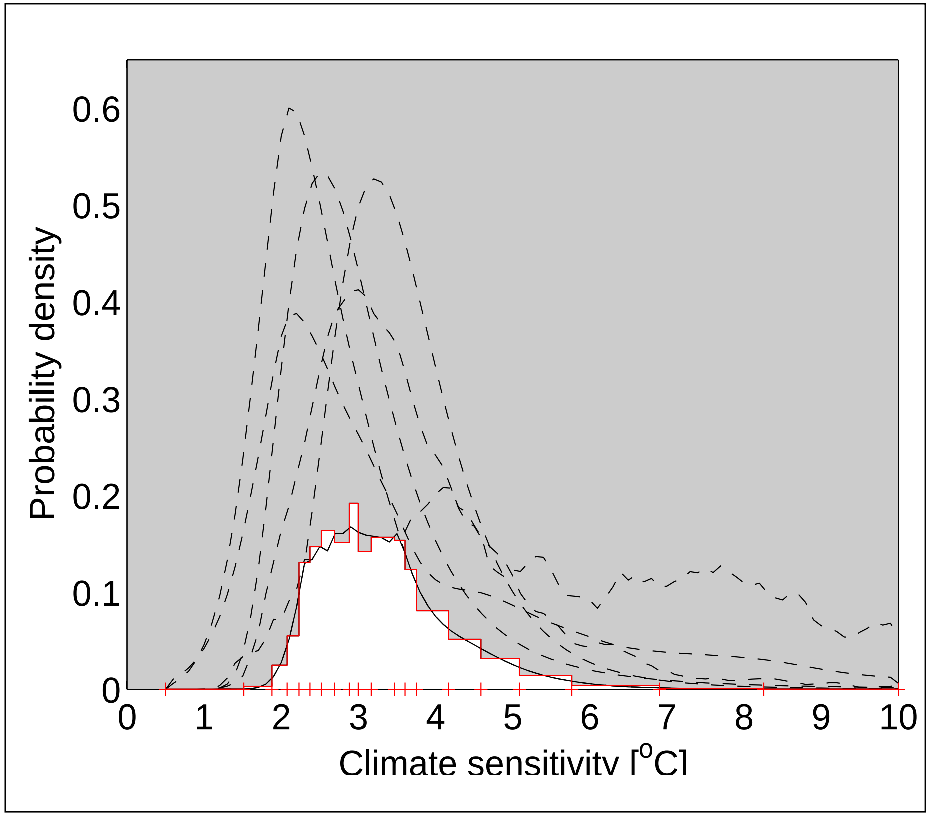
$$\Gamma(\underline{F}, \overline{F}) \equiv \{P \mid \underline{F}(x_j) \le P(-\infty, x_j] \le \overline{F}(x_j)\}$$



Events $(-\infty, x_1], \ldots, (-\infty, x_k]$

Lower mass function class

$$\Gamma(\underline{p}) \equiv \{P \mid \underline{p}_i \le P(A_i), 1 \le i \le n\}$$



Partition $A_1, \ldots, A_n$ of Ω

Class of Imprecise Prior Probability

Intersection of distribution band & lower mass function class

$$\Gamma(\underline{F}, \overline{F}, \underline{p}) \equiv \left\{ P \,\middle|\, \underline{F}(x_j) \leq P(-\infty, x_j] \leq \overline{F}(x_j) \ \wedge\ \underline{p}_i \leq P(A_i),\ 1 \leq j \leq k,\ 1 \leq i \leq n \right\}$$

New algorithm:

Probability bounds on events $\Longrightarrow$ focal elements of Möbius inverse $\mu(\underline{F}, \overline{F}, \underline{p})$

Theorem: Möbius inverse $\mu(\underline{F}, \overline{F}, \underline{p})$ represents the lower envelope $\underline{P}$ of $\Gamma(\underline{F}, \overline{F}, \underline{p})$.

Class of Imprecise Prior Probability

Intersection of distribution band & lower mass function class

$$\Gamma(\underline{F}, \overline{F}, \underline{p}) \equiv \{\ P \mid \underline{F}(x_j) \le P(-\infty, x_j] \le \overline{F}(x_j)\ \wedge\ \underline{p}_i \le P(A_i),\ 1 \le j \le k,\ 1 \le i \le n\ \}$$

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Probability bounds on events $\Longrightarrow$ focal elements of Möbius inverse $\mu(\underline{F}, \overline{F}, \underline{p})$

Theorem: Möbius inverse $\mu(\underline{F}, \overline{F}, \underline{p})$ represents the lower envelope $\underline{P}$ of $\Gamma(\underline{F}, \overline{F}, \underline{p})$.

Möbius inverse is "sparse":

Climate sensitivity: 2^{20} events, 29 focal elements

All parameters $\theta = T_{2x}, \kappa, Q_{S90}$: 2^{3360} events, 6048 focal elements

Generalized Bayesian Updating

Generalized Bayes Rule (GBR)

Updating of all prior probabilities with Bayes' rule

Result: no learning, even loss of information,
since set of prior probabilities too large

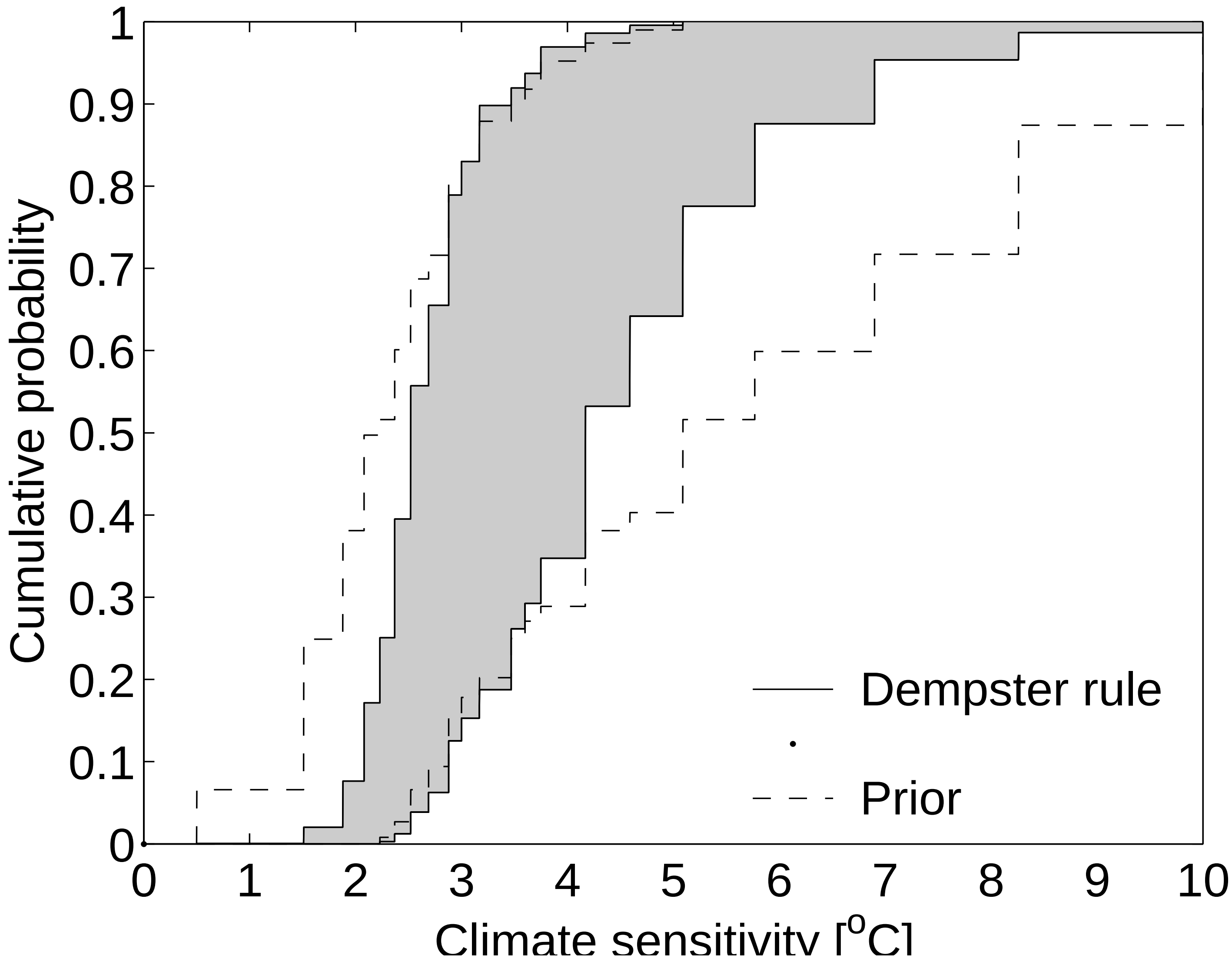
Maximum Likelihood Updating Rule (MLR)

1. Selection of prior probabilities in best agreement with likelihood function

2. Updating of only these prior probabilities with Bayes' rule

Outer bound provided by Dempster's rule for belief function

Posterior Uncertainty about Climate Sensitivity



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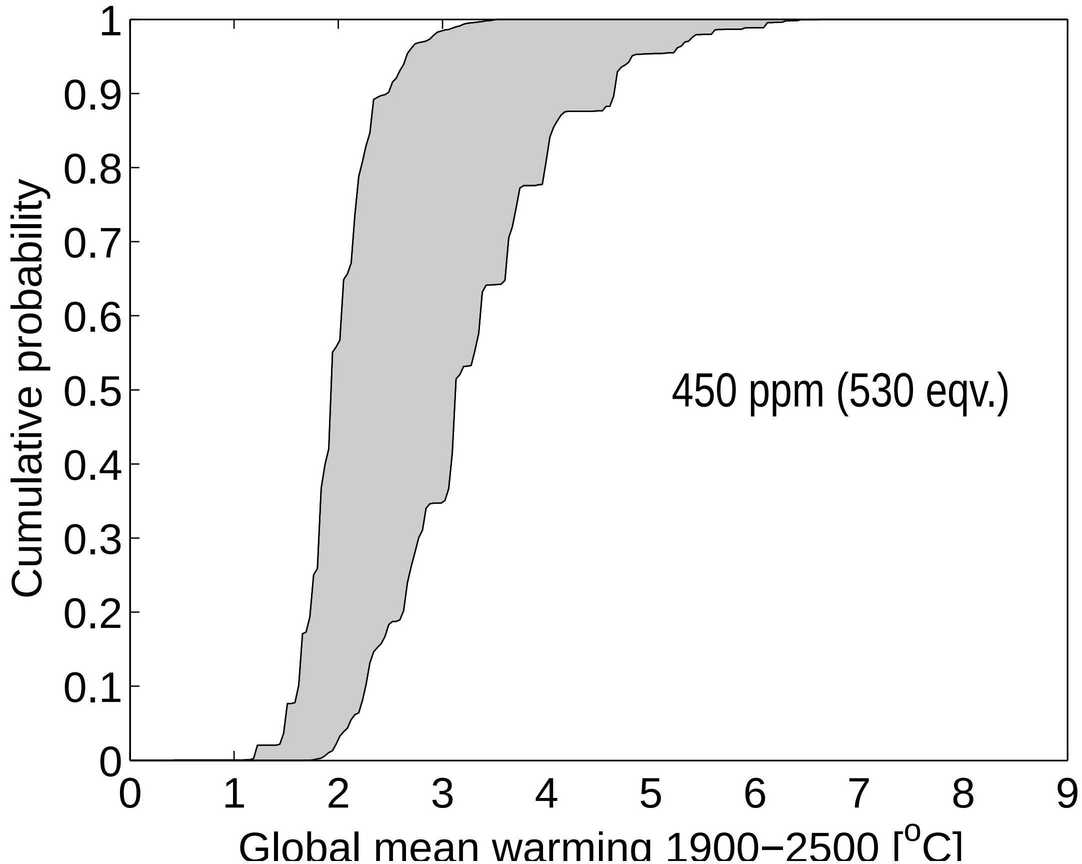
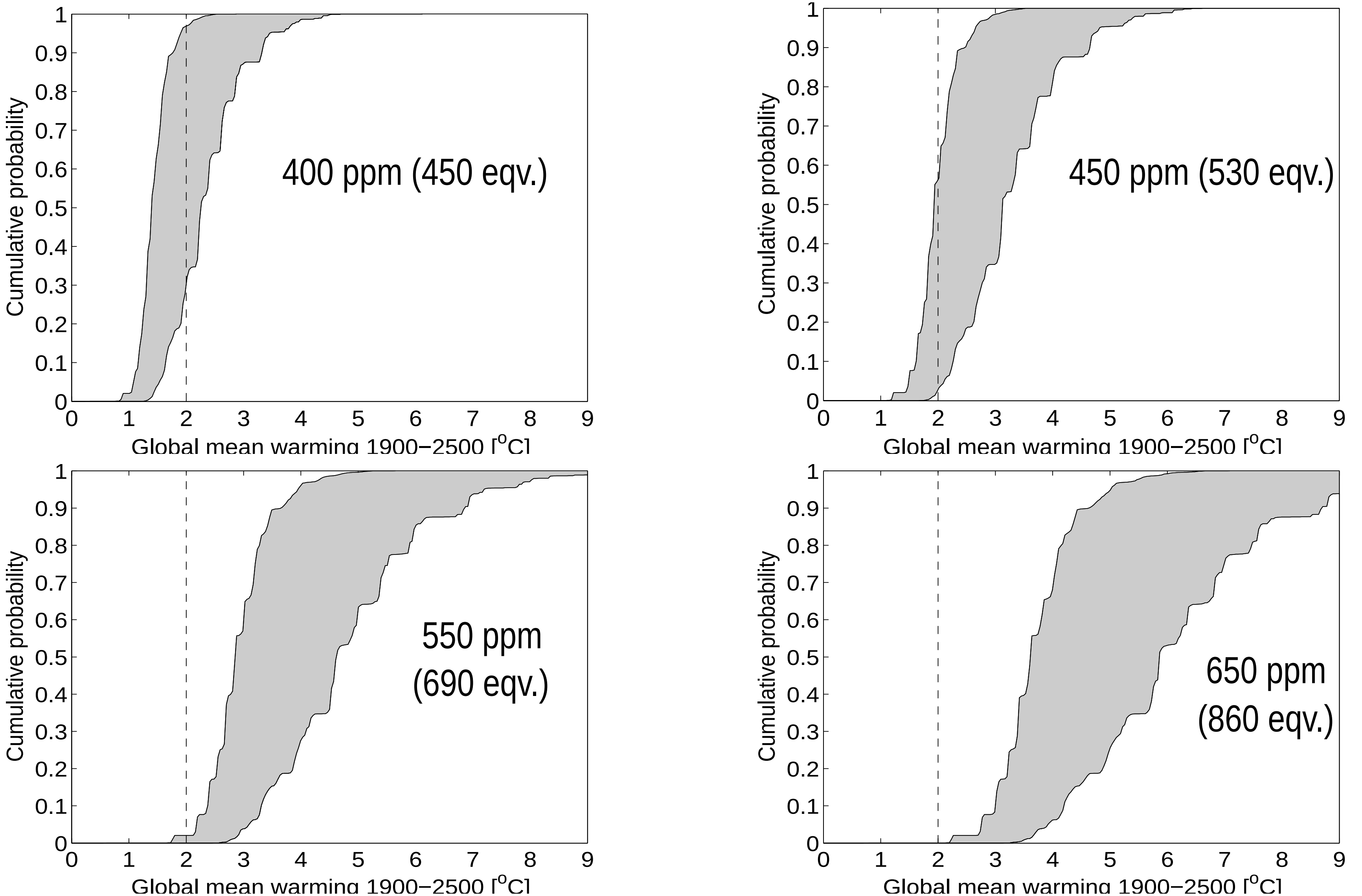
Uncertainty about Future Warming

Projection of posterior belief function (Dempster) for T_{2x}, κ, Q_{S90}

$\Rightarrow$ belief function for global mean temperature increase



Imprecise Estimates for Stabilization Policies



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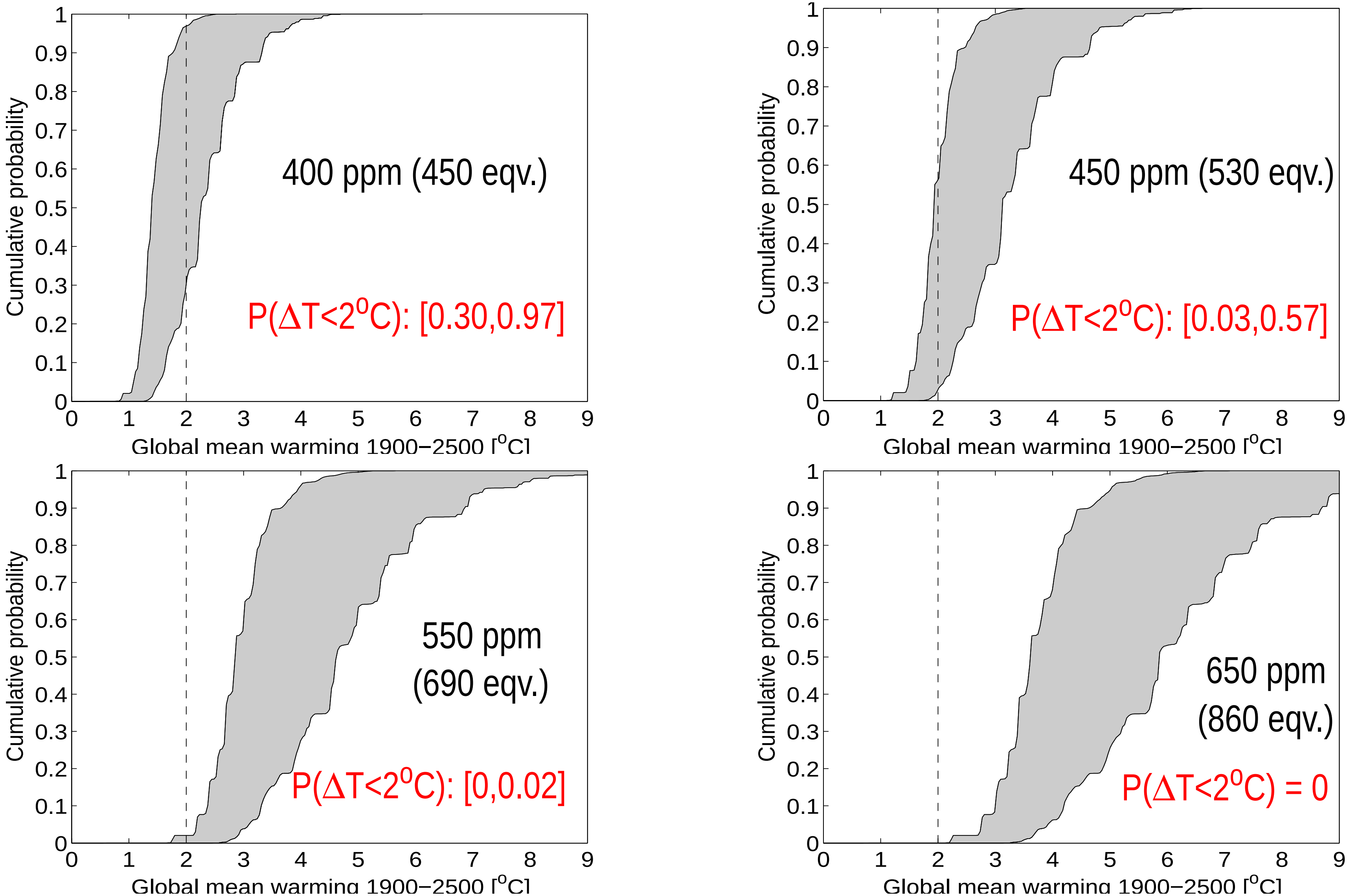
Imprecise Estimates for Stabilization Policies



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What Has Been Achieved?

Application of imprecise probabilities possible?

Yes: Representation in terms of “sparse” Möbius inverses

Improvement for the quantification of the full uncertainty?

Yes: Processing of probability bounds for intuitively accessible events

Possibility to describe very weak states of information

Robust inference of policy relevant quantities possible

What Has Been Achieved?

Application of imprecise probabilities possible?

Yes: Representation in terms of “sparse” Möbius inverses

But: Further tightening of imprecise probability by exclusion of unrealistic priors

Generalization to more complex models

Improvement for the quantification of the full uncertainty?

Yes: Processing of probability bounds for intuitively accessible events

Possibility to describe very weak states of information

Robust inference of policy relevant quantities possible

But: More experience has to be collected with

- estimation of prior probability bounds (e.g., by means of expert elicitations)
- application in a climate policy context

Research Plans as Marie Curie Fellow

Carnegie Mellon: February 2006 - January 2008
based at CDMC, supervised by Granger

Potsdam Institute: February 2008 - January 2009,
based at follow-ups of SPARK project,
supervised by Ottmar Edenhofer & Hermann Held

Current affiliation: SPARK project

Assessment of intertemporal optimal portfolios of mitigation options (energy efficiency, renewables, CCS) under uncertainty

Integrated assessment model MIND (Edenhofer et al., Ecological Economics, in press)

Climate: EBM with 1D-ocean + atmospheric chemistry box model
Economy: Endogenuous growth model with resolved energy sector

Focus on robust strategies under uncertainty

Research Plans as Marie Curie Fellow

- Methodological issues
  Updating, Exclusion of unrealistic priors, Application to expensive models

- Imprecise probabilities and robustification of expert opinions
  Elicitation of probability bounds, Construction of group opinions

- Climate decision making under (imprecise) probability
  Conditions for robust decision criteria under (imprecise) probability

- Policy analysis with MIND under (imprecise) probability
  Key parameters: climate sensitivity, ocean heat uptake, sulfate aerosol forcing
  elasticity of substitution between energy, labour and capital,
  learning rates in renewable energy sector, CCS parameters,
  fossil resource base, return on energy efficiency investment

- Robust policy instruments under uncertainty
  Bonds, permits, taxes

Research Plans as Marie Curie Fellow ...

... need to be flexible to respond to new ideas and circumstances.

I am looking forward to your suggestions for possible contributions to the CDMC during my stay at CMU!

Reach me at: kriegler@pik-potsdam.de